The Nucleon and Delta Baryons in the Unified Picture for Hadron Spectra

A. A. Arkhipov[1]

State Research Center "Institute for High Energy Physics" 142281 Protvino, Moscow Region, Russia

Abstract

In this article it has been shown that the recent PDG Baryon Particle Listings of the Nucleon and Delta states, including some evidence for new states, have excellently incorporated in the unified picture for hadron spectra created earlier [8]. It is claimed that this is a strong confirmation of our theoretical concept. A comparison with experiment is briefly discussed.

1 Introduction

The physics of nucleon resonances has always been a hot matter of discussion in hadronic spectroscopy. It is well known that conventional scheme of classification, systematics and interpretation of all hadronic states [1] is based on the constituent quark model in its many different versions. In the simplest variant of the constituent quark model constructed in the early years all known lightest hadrons made of just three u, d and s quarks were classified in according to irreducible representations of the $SU(3)_f$ group where mesons were made out of $q\bar{q}$, while baryons were built from qqq. By this way the lowest $q\bar{q}$ meson configuration can be decomposed as $\mathbf{3}\otimes\bar{\mathbf{3}} = \mathbf{8}\oplus\mathbf{1}$, while the lowest qqq baryon configuration can be decomposed $\mathbf{3}\otimes\mathbf{3}\otimes\mathbf{3} = \mathbf{10}\oplus\mathbf{8}\oplus\mathbf{8}\oplus\mathbf{1}$ as well. It is a remarkable fact that octet $(\pi^0, \pi^+, \pi^-, K^0, K^+, \bar{K^0}, K^-, \eta)$ of mesons and octet $(p, n, \Sigma^0, \Sigma^+, \Sigma^-, \Xi^0, \Xi^-, \Lambda)$ and decuplet $(\Delta^0, \Delta^-, \Delta^+, \Delta^{++}, \Sigma^{*0}, \Sigma^{*+}, \Sigma^{*-}, \Xi^{*0}, \Xi^{*-}, \Omega^-)$ of baryons have experimentally been observed. The experimental discovery of the predicted Ω^- hyperon was a shining confirmation of $SU(3)_f$ symmetry and of its important role in the classification and systematics of hadronic states. The addition of the c, b, and t quarks to the above three light quarks extends, in principle, the flavor symmetry to $SU(6)_f$. However, earlier the $SU(6)$ group has been considered as an approximate spin-flavor symmetry for the baryons made from just u, d and s quarks (see e.g. [2]). In that case the baryons are classified by the multiplets arising in the decomposition $\mathbf{6}\otimes\mathbf{6}\otimes\mathbf{6} = \mathbf{56}\oplus\mathbf{70}\oplus\mathbf{70}\oplus\mathbf{20}$. Here, the basic states are $u_\uparrow, u_\downarrow, d_\uparrow, d_\downarrow, s_\uparrow, s_\downarrow$ where $\uparrow$ and $\downarrow$ denote spin up and down. Next, the $SU(6)$ multiplets decompose into $SU(3)_f$ multiplets $\mathbf{56} = {}^4\mathbf{10}\oplus{}^2\mathbf{8}$, $\mathbf{70} = {}^2\mathbf{10}\oplus{}^4\mathbf{8}\oplus{}^2\mathbf{8}\oplus{}^2\mathbf{1}$, $\mathbf{20} = {}^2\mathbf{8}\oplus{}^4\mathbf{1}$, where the superscript $(2S+1)$ represents the total spin S of the quarks for all particles in the given $SU(3)_f$ multiplet. So, the above mentioned baryon's octet containing the nucleon, and the decuplet containing $\Delta(1232)$ belong to one and the same

[1]E-mail: Andrei.Arkhipov@ihep.ru

$SU(6)$ multiplet (56-plet) which might be considered as a lowest state where the orbital angular momenta of all quark pairs are zero. Then the **70** and **20** could refer to the states with nonzero orbital angular momenta of quark pairs or something else. In this case the states with nonzero orbital angular momenta may be classified by $SU(6) \otimes O(3)$ supermultiplets. Even though the $SU(6)$ symmetry is broken by spin dependent interactions the $SU(6) \otimes O(3)$ basis was a suitable one for representing the baryon states. However, here the problem of so called "missing" quark-model states arises, it has no solution so far. Of course, in that case one could imagine some selection rules which are responsible for the fact that many baryons have not been observed. At the same time, many recent experiments have reported the observation narrow structures which cannot be explained by the standard quark-model assignments for baryons as qqq states. This, first of all, concerns the number of narrow baryon structures observed in the missing mass M_X and in the $p\pi^+$ invariant mass distribution in the reaction $pp \to p\pi^+ X$, which cannot be associated with the standard qqq quark configurations [3, 4].[2] Why these baryon states are less massive and so narrow than predicted in quark models is an open question so far. Certainly, this raises a challenge to the theory. The other non-qqq baryons have been observed as sharp structures in the nK^+ and pK^0 invariant mass distribution, now denoted as the Θ^+ baryons [5], and in the $\Xi^-\pi^\pm$, $\bar{\Xi}^+\pi^\pm$ invariant mass spectra [6] as well – all of that was interpreted as candidates for pentaquarks ($qqqq\bar{q}$). Here, it should also be mentioned an experimentally well established evidence for many non-$q\bar{q}$ meson's states (see e.g. discussion in Ref. [10] and references therein) declared often and often as exotics. In fact, exotic simply includes all hadrons which cannot be explained in the framework of the simple valence picture of $q\bar{q}$ for mesons or qqq for baryons. Note, that even though the simple valence picture operates degrees of freedom like QCD fields the valence quarks are not identical to QCD fields.

The recent PDG Baryon Particle Listings contain 22 Nucleon and 22 Delta states which are the excited states of the nucleon observed in a large number of formation and production experiments. The conventional masses, widths and other discrete quantum numbers of the N and Δ resonances in the PDG Baryon Listings have largely been defined from partial-wave analysis of πN scattering data. However, any specific constituent quark models even though with a clear set of dynamical ingredients provide quite an another assignments for the quantum numbers of baryons as qqq states and predict a much richer spectrum of baryon states than has been found in partial-wave analysis of πN scattering data. That is why, many attempts have been undertaken to search for "missing" or "hidden" quark-model states from partial-wave analysis in the production processes of other final states such as $N2\pi$, $N\rho$, $N\eta$, $N\omega$, ΛK, ΣK (see e.g. [7]). Besides, there is a serious problem to translate the results of a constituent quark model calculations and predictions into the standard partial-wave analysis conventions accepted by PDG. That translation cannot be constructed in the framework of a given constituent quark model without some additional assumptions and conventions. At any rate, to make such translation in a more clear way we have to consider pentaquark states even for the usual, non-exotic N and Δ baryons. The consistent dynamical description of pentaquark states is very tedious and hard labour which has not been done yet, if not impossible at all. That sheer drudgery will unlikely done in the near future because the consistent dynamical description of more simple three-quark states though has not been performed so far.

[2] I thank B. Tatischeff for drawing my attention to Ref. 3 caused this study.

Concerning the dynamical content of constituent quark models one could say that any specific quark model even with some "QCD-inspired" improvements is a phenomenological, non-relativistic potential model without a reliable ground in quantum field theory, in particular, in QCD. For instance, the usual used constituent quark mass parameters of about 300 MeV for the light u and d quarks cannot be derived in the framework of the underlying QCD. Of course quarks can move relativistically and this also means that theoretical consistency of a non-relativistic potential model is most likely doubtful. We would like also to point out that constituent quark models with the current quark masses about a few MeV have a serious problem with the value of the nucleon sigma term measured in low energy πN scattering. To resolve the σ-term problem the strong Chiral Symmetry Breaking is needed, and the mechanism for that is not clear so far. Nevertheless, there is understanding that such mechanism might be found in the framework of non-perturbative quantum field theory.

Moreover, there is a hope that lattice computations in QCD with a powerful computers could help us to eliminate all imperfections of non-relativistic quark potential models, if lattice studies can help us at all. Time will show.

Recently a new, very simple and quite general theoretical concept concerning the structure of hadron spectra has been formulated which allowed to construct the global solution of the spectral problem in hadron spectroscopy; see [8] and references therein where some of our previous studies were partially summarized. It has been claimed that existence of the extra dimensions in the spirit of Kaluza and Klein together with some novel dynamical ideas may provide new conceptual issues and quite new scheme of systematics for hadron states to build the unified picture for hadron spectra up. The main advantage of the developed theoretical concept is that all calculated numbers for masses and widths of hadron states do not depend on a special dynamical model but follow from fundamental hypothesis on existence of the extra dimensions with a compact internal extra space. One very important fact has been established in a reliable way: the size of the internal compact extra space determines the global characteristics of the hadron spectra while the masses of the decay products are the fundamental parameters of the compound systems being the elements of the global structure. What is remarkable that all new hadron states experimentally discovered last years have been observed just at the masses predicted in our approach, and those states appeared to be narrow as predicted too. A thorough analysis [9] of many different experiments reported the observation of a new very narrow, manifestly exotic Θ^+ (Q=1, S=1) baryon, with the simplest quark assignment $(uudd\bar{s})$ decaying into nK^+ and pK^0_S, taken together allowed us to claim that many different Θ states have been discovered and all of them were excellently incorporated in the unified picture for hadron spectra developed. This concerns the newly discovered $\Xi^{--}_{3/2}$ baryon with strangeness $S = -2$, isospin I $= \frac{3}{2}$ and a quark content of $(dsds\bar{u})$ [6], now denoted as Φ^{--} by PDG, as well. We have also shown that a large amount of experimental data may excellently be incorporated in the systematics provided by the created unified picture for hadron spectra. In this article we apply our approach to show what place in the unified picture for hadron spectra the N and Δ baryons take up.

2 Understanding the N and Δ baryons in the unified picture for hadron spectra

According to the general, theoretical concept [8] we calculate the Kaluza-Klein tower of KK-excitations for the $N\pi$ system by the formula

$$M_n^{N\pi} = \sqrt{m_N^2 + \frac{n^2}{R^2}} + \sqrt{m_\pi^2 + \frac{n^2}{R^2}}\,, \quad (n = 1, 2, 3, ...), \tag{1}$$

where R is the same fundamental scale established before (see [8] and references therein for the details), $N = (p, n)$, $\pi = (\pi^0, \pi^\pm)$, and the masses of proton, neutron and pions have been taken from PDG. The such built Kaluza-Klein tower is shown in Tables 1–5. For simplicity we have considered one-dimensional compact internal extra space and only diagonal elements of the Kaluza-Klein tower have been presented. The experimental data extracted from PDG [1] and Refs. [3, 4] are shown in Tables 1–5 as well. The data from Refs. [3, 4] only are shown in separate Table 5. As seen all narrow low mass baryons shown in the Tables are in excellent agreement with the calculated KK excitations. The other narrow low mass baryon's structures found in Refs. [3, 4], if any, might be identified with non-diagonal elements of the KK tower for the $N\pi$ system. As a rule, non-diagonal elements of the KK towers are suppressed in reality; see however [8] for the details. We would like to emphasize that the so-called universal internal toroidal extra spaces might be considered as a natural explanation of suppression for non-diagonal elements of the KK towers by conservation low of KK numbers. In other words, an experimental observation of hadronic states corresponding to non-diagonal elements of the KK towers could be considered as an evidence for existence of generic internal compact extra spaces.

Our conservative estimate for the widths of KK excitations looks like

$$\Gamma_n \sim \frac{\alpha}{2} \cdot \frac{n}{R} \sim 0.4 \cdot n\,\mathrm{MeV}, \tag{2}$$

where n is the number of KK excitation, and $\alpha \sim 0.02$, $R^{-1} = 41.48\,\mathrm{MeV}$ are known from our previous studies [8]. This model independent estimate is universal for all of the KK towers, it does not depend on a composition of the compound systems living there. Certainly, some model dependent dynamics might modify this estimate. However, one property of estimate (2) is an absolute evidence for the higher the KK excitation is, the larger is the width of the corresponding compound system. This property has to be fulfilled in any model. The broad peaks in the hadron spectra are interpreted in our approach as an envelope of the narrow KK excitations predicted by the Kaluza-Klein scenario.

The most of the nucleon resonances presented in the PDG Baryon Particle Listings have been extracted from partial wave analysis performed by a few groups: the Carnegie-Mellon Berkeley (CMB) group [11], the Karlsruhe-Helsinki (KH) group [12] and the Kent State University (KSU) group [13] are the most famous among of them. It should also be noted the article [14] where the CMB analysis has significantly been extended with account of a larger data set including the modern data at the moment. In fact, the formalism in Ref. [14] is identical to CMB but the data base used is similar to KSU. We would also like to mention the old paper [15] and review article [16]. Each group has an own "cookery" in preparing the results of the analysis, the performed analyses differ from each other often significantly in the methods and the data sets used to extract

the resonances, that is why, there exist sometimes large enough discrepancies between different groups.

Tables 6-7 compare the results of the KSU, KH and CMB analyses with the values of KK excitations for the $p\pi$ system. The best agreements with the KK excitations values are shown in Tables 6-7 by the bold-face numbers.

We have also calculated the Kaluza-Klein towers of KK-excitations for the $N\rho$, $N\eta$, $N\omega$, ΛK, ΣK systems by the formulae similar to (1), and for the $N2\pi$ system by the formula

$$M_n^{N2\pi} = \sqrt{m_N^2 + \frac{n^2}{R^2}} + \sqrt{m_{\pi^1}^2 + \frac{n^2}{R^2}} + \sqrt{m_{\pi^2}^2 + \frac{n^2}{R^2}}\,, \quad (n = 1, 2, 3, ...), \tag{3}$$

as this has been prescribed in [8]; here, as above, $N = (p, n)$, $\pi^1(\pi^2) = (\pi^0, \pi^\pm)$. These KK towers are shown in Tables 8-15. As above, we have restricted ourselves by the simplest case of one-dimensional compact internal extra space and only diagonal elements of the Kaluza-Klein towers have been presented. The arrangement of the known Nucleon and Delta baryons in the Kaluza-Klein towers is presented in Tables 8-15[3] as well.

3 Discussion of comparison with experiment

As seen from the Tables all experimentally observed Nucleon and Delta baryons including narrow low mass baryons are excellently accommodated within them. We see only one empty cell in the $N\pi$ KK tower corresponding to the $M_{26}^{N\pi}(1517)$ storey (see Tables 1-4), but very probably that this fact relates to our incomplete knowledge of modern experimental data base. Sometimes one and the same storey in the $N\pi$ KK tower is occupied by several baryons with approximately equal masses within errors.

It should be emphasized one remarkable fact: we did not find a place for the $P_{33}(1232)$ baryon in Tables 3-4, the $M_5^{N\pi}$(1208-1212) and $M_6^{N\pi}$(1254-1257) storeys are not comfortable for this state. However, we found that the first $M_1^{N2\pi}$(1222-1232) storey in the $N2\pi$ KK tower (see Tables 8-9) is just the place for the $P_{33}(1232)$ baryon. This means that the $P_{33}(1232)$ baryon may have the true three-body origin; really, the symbol Δ is quite appropriate one to correspond to this fact. The other possibility is to search for the $P_{33}(1232)$ baryon among the non-diagonal elements in the $N\pi$ KK tower. For example,

$$M_{nm}^{p\pi^\pm}(n = 3, m = 6) = 1231.84\,\mathrm{MeV}, \quad M_{nm}^{p\pi^\pm}(n = 7, m = 5) = 1232.17\,\mathrm{MeV},$$

$$M_{nm}^{p\pi^\pm}(n = 11, m = 3) = 1230.33\,\mathrm{MeV}, \quad M_{nm}^{n\pi^\pm}(n = 11, m = 3) = 1231.5\,\mathrm{MeV},$$

$$M_{nm}^{n\pi^0}(n = 3, m = 6) = 1230.9\,\mathrm{MeV}, \quad M_{nm}^{n\pi^0}(n = 7, m = 5) = 1230.87\,\mathrm{MeV},$$

where

$$M_{nm}^{N\pi} = \sqrt{m_N^2 + \frac{n^2}{R^2}} + \sqrt{m_\pi^2 + \frac{m^2}{R^2}}\,, \quad (n, m = 1, 2, 3, ...). \tag{4}$$

Tables 8-9 contain the arrangement of the other Delta baryons in the $N2\pi$ KK tower too.

Obviously, that is noteworthy fact, which we would like to point out here, concerning the P_{11} resonance at 1462±10 MeV extracted in Ref. [13]. This resonance just occupies

[3]Even though the Δ states in the $N\eta$, $N\omega$, and ΛK systems are forbidden by isospin we have listed the Δ baryons in Tables 11-13 for convenience too.

the $M_{10}^{N\pi}$-storey in the $N\pi$ KK tower; see Table 1. Note, that the quark-model calculations for the mass of this resonance give 1405 MeV and 1383 MeV [13] that is in strong disagreement.

The new P_{31} resonance at 1744±36 MeV found in [13] is also incorporated in our approach; see $M_{15}^{N\pi}$-storey in the $N\pi$ KK tower in Table 2. The quark-model calculations for the mass of the P_{31} resonance give 1875 MeV and 1906 MeV [13] which are also in strong disagreement. The new F_{35} resonance at 1752±32 MeV found in [13] may occupy the same $M_{15}^{N\pi}$-storey in the $N\pi$ KK tower.

The third D_{13} resonance at 1804±55 MeV found in [13] has also an own place in the $N\pi$ KK tower and especially in the $N\rho$ and $N\eta$ KK towers; see Tables 10-11. Here, one of the quark-model predictions 1809 MeV for the masses of the D_{13} resonances is in good agreement [13].

The second P_{13} resonance at 1879±17 MeV, the third P_{11} resonance at 1885±30 MeV, and the second F_{15} resonance at 1903±87 MeV found in [13] live on one and the same $M_{17}^{N\pi}$-storey in the $N\pi$ KK tower; see Table 1. These resonances have also comfortable places in the $N\rho$, $N\eta$, $N\omega$, ΛK and ΣK KK towers; see Tables 10-15.

Probably the $M_{18}^{N\pi}$-storey in the $N\pi$ KK tower is not so good place for the S_{31} resonance at 1920±24 MeV, and for the S_{11} resonance at 1928±59 MeV found in [13]. However, the $M_{12}^{\Lambda K}$-storey in the ΛK KK tower (Table 13) is quite suitable for these resonances.

We would like to especially emphasize that the lowest D_{35} resonance at 1956±22 MeV, the third P_{33} resonance at 2014±16 MeV, the second D_{33} resonance at 2057±110 MeV, the lowest F_{17} resonance at 2086±28 MeV, and the high-mass D_{35} resonance at 2171±18 MeV found in [13] are excellently incorporated in our approach; see Tables 1-4,10-15. The masses for all of these resonances do not correspond to the quark-model calculations [13].

It seems the $M_{21}^{N\pi}$-storey in the $N\pi$ KK tower is not so comfortable for the lowest G_{17} resonance at 2127±9 MeV found in [13]. Very probably that this resonance lives together with the N[2100]P_{11} and with the F_{17} resonance at 2086±28 MeV on the same storey, as it's clear from Tables 1-4,10-13. In addition, the same G_{17} resonance at 2168±18 MeV found in [14] excellently corresponds to the $M_{21}^{N\pi}$-storey in the $N\pi$ KK tower, and the S_{31} resonance at 1802±87 MeV extracted in [14] is excellently incorporated in our approach too; see Tables 1-4,10-15. .

We would like to mention too the $S_{11}(1535)$ resonance at 1542±3 MeV (in accordance with $M_5^{N\eta}$; see detail discussion in [14]), the $P_{11}(1440)$ resonance at 1479±80 MeV (in accordance with $M_{10}^{N\pi}$), the $P_{11}(1710)$ resonance at 1699±65 MeV (in accordance with $M_{14}^{N\pi}$), the $P_{11}(2100)$ resonance at 2084±93 MeV (in accordance with $M_{20}^{N\pi}$), the $D_{13}(1520)$ resonance at 1518±3 MeV (in accordance with $M_{11}^{N\pi}$), the $G_{17}(2190)$ resonance at 2168±18 MeV (in accordance with $M_{21}^{N\pi}$), the $S_{31}(1620)$ resonance at 1617±15 MeV (in accordance with $M_{13}^{N\pi}$), the $S_{31}(1900)$ resonance at 1802±87 MeV (in accordance with $M_{16}^{N\pi}$), the $P_{31}(1750)$ resonance at 1721±61 MeV (in accordance with $M_{15}^{N\pi}$), and the $F_{35}(1905)$ resonance at 1873±77 MeV (in accordance with $M_{17}^{N\pi}$) found all in Ref. [14] which are also excellently incorporated in the unified picture for hadron spectra.

4 Summary and conclusion

This work should be considered as a continuation of our previous studies concerning the structure of hadron spectra. We have established that the recent PDG Baryon Particle Listings of Nucleon and Delta states, including some evidence for new states, have excellently incorporated in the theoretical concept developed earlier [8]. In particular, it was shown that new resonances found in Ref. [13], including the P_{31} state at 1744±36, the F_{35} state at 1752±32, and P_{13} state at 1879±17 which did not predicted in the quark-model calculations, have excellently accommodated in the corresponding KK towers. Moreover, the recently reported narrow low mass baryons [3, 4], which cannot, in principle, be explained in conventional quark-models, have found own comfortable places in the corresponding KK towers.

Our predictions concerning the masses of hadron states are model independent, they are related with the fundamental hypothesis on existence of the extra dimensions with a compact internal extra space only. In general, each storey in the KK towers is degenerated, i.e. it may contain several flats for hadron states with the different quantum numbers but with approximately equal masses. In addition, the hadron states with the same quantum numbers may have different masses depending on what KK tower they live in, or in other words depending on what decay mode the hadron states have been observed in. This difference in the masses might be measured in the experiments with a high mass resolution. We have already discussed this non-trivial fact in analysing the SELEX measurements; see details in [10].

It should be especially emphasized that KK excitation corresponding to a certain storey in the given KK tower may have exactly the same quantum numbers which have been extracted from partial wave analysis because the definite KK tower corresponds to the definite decay channel what the given partial wave analysis has been done for.

As mentioned above all KK excitations are very narrow, they have the widths about a few MeV; see Eq. (2). The broad peaks in hadron spectra may appear in our approach as an envelope of the narrow KK excitations predicted by the Kaluza-Klein scenario. We have an idea that an availability of non-diagonal elements in the KK towers might play the crucial role in understanding the broad peaks in hadron spectra. This idea has to be explored in the nearest future.

It is well known that the pole positions extracted form partial wave analyses have the least model dependence compared to other parameters such as widths, (in)elasticity, couplings and so on. Our predictions for the masses of KK excitations are strong, that is wy we have performed at the moment only the comparison of the calculated mass values for the KK excitations with the masses of the N and Δ baryons determined from the partial wave analyses.

In conclusion, we would like once again to claim that all well established N and Δ baryons are excellently incorporated in the created unified picture for hadron spectra. No doubt, new experiments with a higher mass resolution and sensitivity would be very helpful to obtain new, more accurate and more full data of high quality. In that case it would be possible to make a reavaluation of all known data to refit the total baryon spectrum and to remove the existing discrepancies. We expect that such new experiments will be set up in the near future for this goal.

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Table 1. Kaluza-Klein tower of KK excitations for the $p\pi$ system and N baryons.

n	$M_n^{p\pi^\pm}$MeV	$M_n^{p\pi^0}$MeV	N BARYONS (PDG) [St:{$\star$}]	$M_{exp}^{N\pi}$ MeV	
1	1084.79	1080.39		1080.6, 1086.5 MAMI	
2	1104.30	1100.37		1104, 1113 SPES4	
3	1133.48	1130.08		1130.4(8), 1133.7 SPES3	
4	1169.65	1166.72		1166.9, 1170.4 SPES3	
5	1210.92	1208.38		1198, 1202 SPES4	
6	1256.07	1253.85		1251.1 SPES3	
7	1304.35	1302.38		1313, 1322 SPES4	
8	1355.23	1353.48		1347.2 MAMI	
9	1408.38	1406.80		1394 SPES4	
10	1463.54	1462.10	N[1440]P_{11} (1430-1470) [4$\star$]	1462$\pm$10 Manley	92
11	1520.50	1519.18	N[1520]D_{13} (1515-1530) [4$\star$]	1519$\pm$ 4 Hoehler	79
12	1579.11	1577.89	N[1535]S_{11} (1520-1555) [4$\star$]	1550$\pm$40 Cutkosky	80
13	1639.22	1638.09	N[1650]S_{11} (1640-1680) [4$\star$]	1650$\pm$30 Cutkosky	80
			N[1675]D_{15} (1670-1685) [4$\star$]	1679$\pm$8 Hoehler	79
			N[1680]F_{15} (1675-1690) [4$\star$]	1680$\pm$10 Cutkosky	80
14	1700.73	1699.67	N[1700]D_{13} (1650-1750) [3$\star$]	1675$\pm$25 Cutkosky	80
			N[1710]P_{11} (1680-1740) [3$\star$]	1717$\pm$28 Manley	92
			N[1720]P_{13} (1650-1750) [4$\star$]	1710$\pm$20 Hoehler	79
15	1763.52	1762.53	P_{11}	1766$\pm$34 Batinic	95
16	1827.50	1826.57	D_{13}	1804$\pm$55 Manley	92
			N[1900]P_{13} ($\approx$ 1900) [2$\star$]	1879$\pm$17 Manley	92
17	1892.59	1891.71	P_{11}	1885$\pm$30 Manley	92
			F_{15}	1903$\pm$87 Manley	92
18	1958.70	1957.87	N[1990]F_{17} ($\approx$ 1990) [2$\star$]	1970$\pm$50 Cutkosky	80
			S_{11}	1928$\pm$59 Manley	92
19	2025.77	2024.98	N[2000]F_{15} ($\approx$ 2000) [2$\star$]	2025 Ayed	76
			N[2080]D_{13} ($\approx$ 2080) [2$\star$]	2081$\pm$20 Hoehler	79
20	2093.73	2092.98	F_{17}	2086$\pm$28 Manley	92
			N[2090]S_{11} ($\approx$ 2090) [1$\star$]	2180$\pm$80 Cutkosky	80
			N[2100]P_{11} ($\approx$ 2100) [1$\star$]	2050$\pm$20 Hoehler	79
21	2162.52	2161.81	N[2190]G_{17} (2100-2200) [4$\star$]	2168$\pm$18 Vrana	00
			G_{17}	2127$\pm$9 Manley	92
			G_{17}	2140$\pm$40 Hendry	78
			N[2200]D_{15} ($\approx$ 2200) [2$\star$]	2228$\pm$30 Hoehler	79
22	2232.08	2231.39	N[2220]H_{19} (2180-2310) [4$\star$]	2230$\pm$80 Cutkosky	80
			N[2250]G_{19} (2170-2310) [4$\star$]	2250$\pm$80 Cutkosky	80
23	2302.36	2301.70	H_{19}	2300$\pm$100 Hendry	78
24	2373.30	2372.68			
25	2444.88	2444.27			
26	2517.03	2516.45			
27	2589.73	2589.17	N[2600]$I_{1,11}$ (2550-2750) [3$\star$]	2577$\pm$50 Hoehler	79
28	2662.94	2662.40	N[2700]$K_{1,13}$ ($\approx$ 2700) [2$\star$]	2612$\pm$45 Hoehler	79
29	2736.63	2736.10	$I_{1,11}$	2700$\pm$100 Hendry	78
30	2810.76	2810.25			

Table 2. Kaluza-Klein tower of KK excitations for the $n\pi$ system and N baryons.

n	$M_n^{n\pi^0}$MeV	$M_n^{n\pi^\pm}$MeV	N BARYONS (PDG) [St:{$\star$}]		$M_{exp}^{N\pi}$ MeV	
1	1081.69	1086.08			1080.6, 1086.5 MAMI	
2	1101.66	1105.59			1104, 1113 SPES4	
3	1131.36	1134.76			1130.4(8), 1133.7 SPES3	
4	1168.00	1170.92			1166.9, 1170.4 SPES3	
5	1209.64	1212.18			1198, 1202 SPES4	
6	1255.10	1257.32			1251.1 SPES3	
7	1303.62	1305.58			1313, 1322 SPES4	
8	1354.70	1356.45			1347.2 MAMI	
9	1408.00	1409.59			1394 SPES4	
10	1463.28	1464.72	N[1440]P_{11} (1430-1470)	[4$\star$]	1462$\pm$10 Manley	92
11	1520.34	1521.67	N[1520]D_{13} (1515-1530)	[4$\star$]	1519$\pm$4 Hoehler	79
12	1579.03	1580.25	N[1535]S_{11} (1520-1555)	[4$\star$]	1550$\pm$40 Cutkosky	80
13	1639.21	1640.34	N[1650]S_{11} (1640-1680)	[4$\star$]	1650$\pm$30 Cutkosky	80
			N[1675]D_{15} (1670-1685)	[4$\star$]	1679$\pm$8 Hoehler	79
			N[1680]F_{15} (1675-1690)	[4$\star$]	1680$\pm$10 Cutkosky	80
14	1700.77	1701.83	N[1700]D_{13} (1650-1750)	[3$\star$]	1675$\pm$25 Cutkosky	80
			N[1710]P_{11} (1680-1740)	[3$\star$]	1717$\pm$28 Manley	92
			N[1720]P_{13} (1650-1750)	[4$\star$]	1710$\pm$20 Hoehler	79
15	1763.61	1764.60	P_{11}		1766$\pm$34 Batinic	95
16	1827.62	1828.56	D_{13}		1804$\pm$55 Manley	92
			N[1900]P_{13} ($\approx$ 1900)	[2$\star$]	1879$\pm$17 Manley	92
17	1892.74	1893.62	P_{11}		1885$\pm$30 Manley	92
			F_{15}		1903$\pm$87 Manley	92
18	1958.88	1959.71	N[1990]F_{17} ($\approx$ 1990)	[2$\star$]	1970$\pm$50 Cutkosky	80
			S_{11}		1928$\pm$59 Manley	92
19	2025.98	2026.76	N[2000]F_{15} ($\approx$ 2000)	[2$\star$]	2025 Ayed	76
			N[2080]D_{13} ($\approx$ 2080)	[2$\star$]	2081$\pm$20 Hoehler	79
20	2093.95	2094.70	F_{17}		2086$\pm$28 Manley	92
			N[2090]S_{11} ($\approx$ 2090)	[1$\star$]	2180$\pm$80 Cutkosky	80
			N[2100]P_{11} ($\approx$ 2100)	[1$\star$]	2050$\pm$20 Hoehler	79
21	2162.75	2163.47	N[2190]G_{17} (2100-2200)	[4$\star$]	2168$\pm$18 Vrana	00
			G_{17}		2127$\pm$9 Manley	92
			G_{17}		2140$\pm$40 Hendry	78
			N[2200]D_{15} ($\approx$ 2200)	[2$\star$]	2228$\pm$30 Hoehler	79
22	2232.32	2233.01	N[2220]H_{19} (2180-2310)	[4$\star$]	2230$\pm$80 Cutkosky	80
			N[2250]G_{19} (2170-2310)	[4$\star$]	2250$\pm$80 Cutkosky	80
23	2302.61	2303.26	H_{19}		2300$\pm$100 Hendry	78
24	2373.56	2374.19				
25	2445.14	2445.74				
26	2517.30	2517.88				
27	2590.00	2590.56	N[2600]$I_{1,11}$ (2550-2750)	[3$\star$]	2577$\pm$50 Hoehler	79
28	2663.21	2663.75	N[2700]$K_{1,13}$ ($\approx$ 2700)	[2$\star$]	2612$\pm$45 Hoehler	79
29	2736.90	2737.42	$I_{1,11}$		2700$\pm$100 Hendry	78
30	2811.03	2811.54				

Table 3. Kaluza-Klein tower of KK excitations for the $p\pi$ system and Δ baryons.

n	$M_n^{p\pi^\pm}$ MeV	$M_n^{p\pi^0}$ MeV	Δ BARYONS (PDG) [St:{$\star$}]	$M_{exp}^{N\pi}$ MeV
1	1084.79	1080.39		
2	1104.30	1100.37		
3	1133.48	1130.08		
4	1169.65	1166.72		
5	1210.92	1208.38		
6	1256.07	1253.85		
7	1304.35	1302.38		
8	1355.23	1353.48		
9	1408.38	1406.80		
10	1463.54	1462.10		
11	1520.50	1519.18		
12	1579.11	1577.89	$\Delta[1600]P_{33}$ (1550-1700) [3$\star$]	1600$\pm$50 Cutkosky 80
13	1639.22	1638.09	$\Delta[1620]S_{31}$ (1615-1675) [4$\star$]	1620$\pm$20 Cutkosky 80
14	1700.73	1699.67	$\Delta[1700]D_{33}$ (1670-1770) [4$\star$]	1710$\pm$30 Cutkosky 80
15	1763.52	1762.53	$\Delta[1750]P_{31}$ ($\approx$ 1750) [1$\star$]	1744$\pm$36 Manley 92
			F_{35}	1752$\pm$32 Manley 92
16	1827.50	1826.57	S_{31}	1802$\pm$87 Vrana 00
			$\Delta[1900]S_{31}$ (1850-1950) [2$\star$]	1890$\pm$50 Cutkosky 80
17	1892.59	1891.71	$\Delta[1905]F_{35}$ (1870-1920) [4$\star$]	1881$\pm$18 Manley 92
			$\Delta[1910]P_{31}$ (1870-1920) [4$\star$]	1888$\pm$20 Hoehler 79
			S_{31}	1920$\pm$24 Manley 92
			$\Delta[1920]P_{33}$ (1900-1970) [3$\star$]	1920$\pm$80 Cutkosky 80
18	1958.70	1957.87	$\Delta[1930]D_{35}$ (1920-1970) [3$\star$]	1956$\pm$22 Manley 92
			$\Delta[1940]D_{33}$ ($\approx$ 1940) [1$\star$]	1940$\pm$100 Cutkosky 80
			$\Delta[1950]F_{37}$ (1940-1960) [4$\star$]	1950$\pm$15 Cutkosky 80
19	2025.77	2024.98	$\Delta[2000]F_{35}$ ($\approx$ 2000) [2$\star$]	1752$\pm$32 Manley 92
			P_{33}	2014$\pm$16 Manley 92
20	2093.73	2092.98	D_{33}	2057$\pm$110 Manley 92
21	2162.52	2161.81	$\Delta[2150]S_{31}$ ($\approx$ 2150) [1$\star$]	2150$\pm$100 Cutkosky 80
			D_{35}	2171$\pm$18 Manley 92
22	2232.08	2231.39	$\Delta[2200]G_{37}$ ($\approx$ 2200) [1$\star$]	2215$\pm$60 Hoehler 79
23	2302.36	2301.70	$\Delta[2300]H_{39}$ ($\approx$ 2300) [2$\star$]	2217$\pm$80 Hoehler 79
24	2373.30	2372.68	$\Delta[2350]D_{35}$ ($\approx$ 2350) [2$\star$]	2305$\pm$26 Hoehler 79
			$\Delta[2390]F_{37}$ ($\approx$ 2390) [1$\star$]	2350$\pm$100 Cutkosky 80
			H_{39}	2450$\pm$100 Hendry 78
25	2444.88	2444.27	$\Delta[2400]G_{39}$ ($\approx$ 2400) [2$\star$]	2468$\pm$50 Hoehler 79
			$\Delta[2420]H_{3,11}$ (2300-2500) [4$\star$]	2416$\pm$17 Hoehler 79
26	2517.03	2516.45		
27	2589.73	2589.17		
28	2662.94	2662.40	$I_{3,13}$	2650$\pm$100 Hendry 78
29	2736.63	2736.10	$\Delta[2750]I_{3,13}$ ($\approx$ 2750) [2$\star$]	2794$\pm$80 Hoehler 79
30	2810.76	2810.25	$\Delta[2950]K_{3,15}$ ($\approx$ 2950) [2$\star$]	2850$\pm$100 Hendry 78

Table 4. Kaluza-Klein tower of KK excitations for the $n\pi$ system and Δ baryons.

n	$M_n^{n\pi^0}$ MeV	$M_n^{n\pi^\pm}$ MeV	Δ BARYONS (PDG) [St:{$\star$}]	$M_{exp}^{N\pi}$ MeV
1	1081.69	1086.08		
2	1101.66	1105.59		
3	1131.36	1134.76		
4	1168.00	1170.92		
5	1209.64	1212.18		
6	1255.10	1257.32		
7	1303.62	1305.58		
8	1354.70	1356.45		
9	1408.00	1409.59		
10	1463.28	1464.72		
11	1520.34	1521.67		
12	1579.03	1580.25	$\Delta[1600]P_{33}$ (1550-1700) [3$\star$]	1600±50 Cutkosky 80
13	1639.21	1640.34	$\Delta[1620]S_{31}$ (1615-1675) [4$\star$]	1620±20 Cutkosky 80
14	1700.77	1701.83	$\Delta[1700]D_{33}$ (1670-1770) [4$\star$]	1710±30 Cutkosky 80
15	1763.61	1764.60	$\Delta[1750]P_{31}$ ($\approx$ 1750) [1$\star$]	1744±36 Manley 92
			F_{35}	1752±32 Manley 92
16	1827.62	1828.56	S_{31}	1802±87 Vrana 00
			$\Delta[1900]S_{31}$ (1850-1950) [2$\star$]	1890±50 Cutkosky 80
17	1892.74	1893.62	$\Delta[1905]F_{35}$ (1870-1920) [4$\star$]	1881±18 Manley 92
			$\Delta[1910]P_{31}$ (1870-1920) [4$\star$]	1888±20 Hoehler 79
			S_{31}	1920±24 Manley 92
			$\Delta[1920]P_{33}$ (1900-1970) [3$\star$]	1920±80 Cutkosky 80
18	1958.88	1959.71	$\Delta[1930]D_{35}$ (1920-1970) [3$\star$]	1956±22 Manley 92
			$\Delta[1940]D_{33}$ ($\approx$ 1940) [1$\star$]	1940±100 Cutkosky 80
			$\Delta[1950]F_{37}$ (1940-1960) [4$\star$]	1950±15 Cutkosky 80
19	2025.98	2026.76	$\Delta[2000]F_{35}$ ($\approx$ 2000) [2$\star$]	1752±32 Manley 92
			P_{33}	2014±16 Manley 92
20	2093.95	2094.70	D_{33}	2057±110 Manley 92
21	2162.75	2163.47	$\Delta[2150]S_{31}$ ($\approx$ 2150) [1$\star$]	2150±100 Cutkosky 80
			D_{35}	2171±18 Manley 92
22	2232.32	2233.01	$\Delta[2200]G_{37}$ ($\approx$ 2200) [1$\star$]	2215±60 Hoehler 79
23	2302.61	2303.26	$\Delta[2300]H_{39}$ ($\approx$ 2300) [2$\star$]	2217±80 Hoehler 79
24	2373.56	2374.19	$\Delta[2350]D_{35}$ ($\approx$ 2350) [2$\star$]	2305±26 Hoehler 79
			$\Delta[2390]F_{37}$ ($\approx$ 2390) [1$\star$]	2350±100 Cutkosky 80
			H_{39}	2450±100 Hendry 78
25	2445.14	2445.74	$\Delta[2400]G_{39}$ ($\approx$ 2400) [2$\star$]	2468±50 Hoehler 79
			$\Delta[2420]H_{3,11}$ (2300-2500) [4$\star$]	2416±17 Hoehler 79
26	2517.30	2517.88		
27	2590.00	2590.56		
28	2663.21	2663.75	$I_{3,13}$	2650±100 Hendry 78
29	2736.90	2737.42	$\Delta[2750]I_{3,13}$ ($\approx$ 2750) [2$\star$]	2794±80 Hoehler 79
30	2811.03	2811.54	$\Delta[2950]K_{3,15}$ ($\approx$ 2950) [2$\star$]	2850±100 Hendry 78

Table 5. KK excitations for the $N\pi$ system and narrow exotic low mass baryons [3, 4].

n	$M_n^{p\pi^\pm}$MeV	$M_n^{p\pi^0}$MeV	$M_n^{n\pi^0}$MeV	$M_n^{n\pi^\pm}$MeV	$M_{exp}^{N\pi}$ MeV
1	1084.79	1080.39	1081.69	1086.08	1080.6, 1086.5 MAMI
2	1104.30	1100.37	1101.66	1105.59	1104, 1113 SPES4
3	1133.48	1130.08	1131.36	1134.76	1130.4(8), 1133.7 SPES3
4	1169.65	1166.72	1168.00	1170.92	1166.9, 1170.4 SPES3
5	1210.92	1208.38	1209.64	1212.18	1198, 1202 SPES4
6	1256.07	1253.85	1255.10	1257.32	1251.1 SPES3
7	1304.35	1302.38	1303.62	1305.58	1313, 1322 SPES4
8	1355.23	1353.48	1354.70	1356.45	1347.2 MAMI
9	1408.38	1406.80	1408.00	1409.59	1394 SPES4
10	1463.54	1462.10	1463.28	1464.72	1477 SPES4
11	1520.50	1519.18	1520.34	1521.67	1517 SPES3 SPES4
12	1579.11	1577.89	1579.03	1580.25	1577 SPES4
13	1639.22	1638.09	1639.21	1640.34	1639 SPES4
14	1700.73	1699.67	1700.77	1701.83	
15	1763.52	1762.53	1763.61	1764.60	
16	1827.50	1826.57	1827.62	1828.56	
17	1892.59	1891.71	1892.74	1893.62	
18	1958.70	1957.87	1958.88	1959.71	
19	2025.77	2024.98	2025.98	2026.76	
20	2093.73	2092.98	2093.95	2094.70	
21	2162.52	2161.81	2162.75	2163.47	
22	2232.08	2231.39	2232.32	2233.01	
23	2302.36	2301.70	2302.61	2303.26	
24	2373.30	2372.68	2373.56	2374.19	
25	2444.88	2444.27	2445.14	2445.74	
26	2517.03	2516.45	2517.30	2517.88	
27	2589.73	2589.17	2590.00	2590.56	
28	2662.94	2662.40	2663.21	2663.75	
29	2736.63	2736.10	2736.90	2737.42	
30	2810.76	2810.25	2811.03	2811.54	

Table 6. Kaluza-Klein tower of KK excitations for the $p\pi$ system compared with the masses of the N baryons determined from the KSU [13], KH [12] and CMB [11] analyses.

n	$M_n^{p\pi^\pm}$MeV	$M_n^{p\pi^0}$MeV	N(PDG) [St:{$\star$}]	KSU	KH	CMB
1	1084.79	1080.39				
2	1104.30	1100.37				
3	1133.48	1130.08				
4	1169.65	1166.72				
5	1210.92	1208.38				
6	1256.07	1253.85				
7	1304.35	1302.38				
8	1355.23	1353.48				
9	1408.38	1406.80				
10	1463.54	1462.10	N[1440]P_{11} [4$\star$]	**1462±10**	1410±12	1440±30
11	1520.50	1519.18	N[1520]D_{13} [4$\star$]	1524±4	**1519±4**	1525±10
12	1579.11	1577.89	N[1535]S_{11} [4$\star$]	1534±7	1526±7	**1550±40**
13	1639.22	1638.09	N[1650]S_{11} [4$\star$]	1659±9	1670±8	**1650±30**
			N[1675]D_{15} [4$\star$]	1676±2	1679±8	1675±10
			N[1680]F_{15} [4$\star$]	1684±4	1684±3	1680±10
14	1700.73	1699.67	N[1700]D_{13} [3$\star$]	1737±44	1731±15	1675±25
			N[1710]P_{11} [3$\star$]	1717±28	1723±9	**1700±50**
			N[1720]P_{13} [4$\star$]	1717±31	1710±20	**1700±50**
15	1763.52	1762.53				
16	1827.50	1826.57	D_{13}	1804±55		
			N[1900]P_{13} [2$\star$]	1879±17		
17	1892.59	1891.71	P_{11}	**1885±30**		
			F_{15}	1903±87		
18	1958.70	1957.87	N[1990]F_{17} [2$\star$]	2086±28	2005±150	**1970±50**
			S_{11}	1928±59		
19	2025.77	2024.98	N[2000]F_{15} [2$\star$]	1903±97	1882±10	
			N[2080]D_{13} [2$\star$]	1804±55	2081±20	2060±80
20	2093.73	2092.98	F_{17}	**2086±28**		
			N[2090]S_{11} [1$\star$]	1928±59	1880±20	2180±80
			N[2100]P_{11} [1$\star$]		2050±20	
21	2162.52	2161.81	N[2190]G_{17} [4$\star$]	2127±9	**2140±12**	2200±70
			N[2200]D_{15} [2$\star$]		2228±30	
22	2232.08	2231.39	N[2220]H_{19} [4$\star$]			**2230±80**
			N[2250]G_{19} [4$\star$]		2250±80	
23	2302.36	2301.70				
24	2373.30	2372.68				
25	2444.88	2444.27				
26	2517.03	2516.45				
27	2589.73	2589.17	N[2600]$I_{1,11}$ [3$\star$]		**2577±50**	
28	2662.94	2662.40	N[2700]$K_{1,13}$ [2$\star$]		2612±45	
29	2736.63	2736.10				
30	2810.76	2810.25				

Table 7. Kaluza-Klein tower of KK excitations for the $p\pi$ system compared with the masses of the Δ baryons determined from the KSU [13], KH [12] and CMB [11] analyses.

n	$M_n^{p\pi^\pm}$MeV	$M_n^{p\pi^0}$MeV	Δ(PDG) [St:{$\star$}]	KSU	KH	CMB
1	1084.79	1080.39				
2	1104.30	1100.37				
3	1133.48	1130.08				
4	1169.65	1166.72				
5	1210.92	1208.38				
6	1256.07	1253.85				
7	1304.35	1302.38				
8	1355.23	1353.48				
9	1408.38	1406.80				
10	1463.54	1462.10				
11	1520.50	1519.18				
12	1579.11	1577.89	$\Delta[1600]P_{33}$ [3$\star$]	1706±10	1522±13	**1600±50**
13	1639.22	1638.09	$\Delta[1620]S_{31}$ [4$\star$]	1672±7	1610±7	**1620±20**
14	1700.73	1699.67	$\Delta[1700]D_{33}$ [4$\star$]	1762±44	1680±70	**1710±30**
15	1763.52	1762.53	$\Delta[1750]P_{31}$ [1$\star$]	1744±36		
			F_{35}	**1752±32**		
16	1827.50	1826.57				
			$\Delta[1900]S_{31}$ [2$\star$]	1920±24	1908±30	**1890±50**
17	1892.59	1891.71	$\Delta[1905]F_{35}$ [4$\star$]	1881±18	1905±20	1910±30
			$\Delta[1910]P_{31}$ [4$\star$]	1882±10	**1888±20**	1910±40
			S_{31}	1920±24		
			$\Delta[1920]P_{33}$ [3$\star$]	2014±16	1868±10	1920±80
18	1958.70	1957.87	$\Delta[1930]D_{35}$ [3$\star$]	**1956±22**	1901±15	1940±30
			$\Delta[1940]D_{33}$ [1$\star$]	2057±110		1940±100
			$\Delta[1950]F_{37}$ [4$\star$]	1945±2	1913±8	1950±15
19	2025.77	2024.98	$\Delta[2000]F_{35}$ [2$\star$]	1752±32		2200±125
			P_{33}	**2014±16**		
20	2093.73	2092.98	D_{33}	**2057±110**		
21	2162.52	2161.81	$\Delta[2150]S_{31}$ [1$\star$]			2150±100
			D_{35}	**2171±18**		
22	2232.08	2231.39	$\Delta[2200]G_{37}$ [1$\star$]		**2215±60**	
23	2302.36	2301.70	$\Delta[2300]H_{39}$ [2$\star$]		2217±80	
24	2373.30	2372.68	$\Delta[2350]D_{35}$ [2$\star$]		2305±26	2400±125
			$\Delta[2390]F_{37}$ [1$\star$]			**2350±100**
25	2444.88	2444.27	$\Delta[2400]G_{39}$ [2$\star$]		**2468±50**	
			$\Delta[2420]H_{3,11}$ [4$\star$]		2416±17	
26	2517.03	2516.45				
27	2589.73	2589.17				
28	2662.94	2662.40				
29	2736.63	2736.10	$\Delta[2750]I_{3,13}$ [2$\star$]		**2794±80**	
30	2810.76	2810.25				

Table 8. Kaluza-Klein tower of KK excitations for the $p\pi\pi$ system and Δ baryons.

n	$M_n^{p\,2\pi^0}$MeV	$M_n^{p\,\pi^0\pi^\pm}$MeV	$M_n^{p\,2\pi^\pm}$MeV	Δ BARYON	M_{exp}^{Δ} MeV
1	1221.60	1226.00	1230.40	$\Delta[1232]$ P_{33}	1231±1 Manley 92
2	1258.80	1262.73	1266.66		
3	1313.67	1317.07	1320.47		
4	1380.61	1383.54	1386.47		
5	1455.84	1458.38	1460.91		
6	1536.98	1539.20	1541.42	$\Delta[1600]$ P_{33}	1522±13 Hoehler 79
7	1622.59	1624.55	1626.52	$\Delta[1620]$ S_{31}	1620±20 Cutkosky 80
8	1711.73	1713.48	1715.24	$\Delta[1700]$ D_{33}	1710±30 Cutkosky 80
9	1803.78	1805.37	1806.95	S_{31}	1802±87 Vrana 00
				$\Delta[1900]$ S_{31}	1890±50 Cutkosky 80
10	1898.32	1899.76	1901.20	$\Delta[1905]$ F_{35}	1881±18 Manley 92
				$\Delta[1910]$ P_{31}	1888±20 Hoehler 79
11	1995.02	1996.34	1997.66	$\Delta(1910)$ P_{31}	1995±12 Vrana 00
				$\Delta[1920]$ P_{33}	1920±80 Cutkosky 80
				$\Delta[1930]$ D_{35}	1956±22 Manley 92
				$\Delta[1940]$ D_{33}	1940±100 Cutkosky 80
				$\Delta[1950]$ F_{37}	1950±15 Cutkosky 80
				$\Delta[2000]$ F_{35}	1752±32 Manley 92
12	2093.64	2094.86	2096.08	$\Delta[2150]$ S_{31}	2150±100 Cutkosky 80
				P_{33}	$2065^{+13.6}_{-12.9}$ Chew 80
13	2193.98	2195.11	2196.25	$\Delta[2150]$ S_{31}	2150±100 Cutkosky 80
				$\Delta[2200]$ G_{37}	2215±60 Hoehler 79
14	2295.89	2296.94	2298.00	$\Delta[2300]$ H_{39}	2217±80 Hoehler 79
				$\Delta[2350]$ D_{35}	2305±26 Hoehler 79
				$\Delta[2390]$ F_{37}	2350±100 Cutkosky 80
15	2399.22	2400.21	2401.20	$\Delta[2400]$ G_{39}	2468±50 Hoehler 79
				$\Delta[2420]H_{3,11}$	2416±17 Hoehler 79
16	2503.85	2504.78	2505.72		
17	2609.69	2610.57	2611.45	$\Delta[2750]I_{3,13}$	2650±100 Hendry 78
18	2716.64	2717.47	2718.30	$\Delta[2750]I_{3,13}$	2794±80 Hoehler 79
19	2824.60	2825.39	2826.18	$\Delta[2950]K_{3,15}$	2850±100 Hendry 78
20	2933.52	2934.27	2935.02	$\Delta[2950]K_{3,15}$	2990±100 Hoehler 79
21	3043.31	3044.02	3044.74		
22	3153.91	3154.59	3155.28	$K_{3,13}$	3200±200 Hendry 78
23	3265.27	3265.92	3266.58	$L_{3,17}$	3300±200 Hendry 78
24	3377.33	3377.96	3378.59		
25	3490.05	3490.65	3491.26		
26	3603.38	3603.96	3604.54		
27	3717.27	3717.83	3718.39	$M_{3,19}$	3700±200 Hendry 78
28	3831.69	3832.23	3832.77		
29	3946.61	3947.13	3947.65		
30	4061.99	4062.49	4063.00	$N_{3,21}$	4100±300 Hendry 78

Table 9. Kaluza-Klein tower of KK excitations for the $n\pi\pi$ system and Δ baryons.

n	$M_n^{n2\pi^0}$MeV	$M_n^{n\,\pi^0\pi^\pm}$MeV	$M_n^{n2\pi^\pm}$MeV	Δ BARYON	M_{exp}^{Δ} MeV	
1	1222.89	1227.29	1231.69	$\Delta[1232]\ P_{33}$	1231±1 Manley	92
2	1260.09	1264.02	1267.95			
3	1314.95	1318.35	1321.75			
4	1381.89	1384.82	1387.74			
5	1457.10	1459.64	1462.17			
6	1538.23	1540.45	1542.67	$\Delta[1600]\ P_{33}$	1522±13 Hoehler	79
7	1623.82	1625.79	1627.75	$\Delta[1620]\ S_{31}$	1620±20 Cutkosky	80
8	1712.95	1714.70	1716.46	$\Delta[1700]\ D_{33}$	1710±30 Cutkosky	80
9	1804.98	1806.57	1808.15	S_{31}	1802±87 Vrana	00
				$\Delta[1900]\ S_{31}$	1890±50 Cutkosky	80
10	1899.50	1900.94	1902.39	$\Delta[1905]\ F_{35}$	1881±18 Manley	92
				$\Delta[1910]\ P_{31}$	1888±20 Hoehler	79
11	1996.18	1997.50	1998.83	$\Delta(1910)\ P_{31}$	1995±12 Vrana	00
				$\Delta[1920]\ P_{33}$	1920±80 Cutkosky	80
				$\Delta[1930]\ D_{35}$	1956±22 Manley	92
				$\Delta[1940]\ D_{33}$	1940±100 Cutkosky	80
				$\Delta[1950]\ F_{37}$	1950±15 Cutkosky	80
				$\Delta[2000]\ F_{35}$	1752±32 Manley	92
12	2093.64	2094.86	2096.08	$\Delta[2150]\ S_{31}$	2150±100 Cutkosky	80
				P_{33}	$2065^{+13.6}_{-12.9}$ Chew	80
13	2193.98	2195.11	2196.25	$\Delta[2150]\ S_{31}$	2150±100 Cutkosky	80
				$\Delta[2200]\ G_{37}$	2215±60 Hoehler	79
14	2296.99	2298.04	2299.10	$\Delta[2300]\ H_{39}$	2217±80 Hoehler	79
				$\Delta[2350]\ D_{35}$	2305±26 Hoehler	79
				$\Delta[2390]\ F_{37}$	2350±100 Cutkosky	80
15	2400.30	2401.29	2402.28	$\Delta[2400]\ G_{39}$	2468±50 Hoehler	79
				$\Delta[2420] H_{3,11}$	2416±17 Hoehler	79
16	2504.91	2505.84	2506.77			
17	2610.73	2611.60	2612.48	$\Delta[2750] I_{3,13}$	2650±100 Hendry	78
18	2717.65	2718.49	2719.31	$\Delta[2750] I_{3,13}$	2794±80 Hoehler	79
19	2825.59	2826.38	2827.17	$\Delta[2950] K_{3,15}$	2850±100 Hendry	78
20	2934.48	2935.24	2935.98	$\Delta[2950] K_{3,15}$	2990±100 Hoehler	79
21	3044.25	3044.97	3045.68			
22	3154.84	3155.52	3156.20	$K_{3,13}$	3200±200 Hendry	78
23	3266.18	3266.83	3267.49	$L_{3,17}$	3300±200 Hendry	78
24	3378.22	3378.85	3379.48			
25	3490.92	3491.52	3492.12			
26	3604.22	3604.80	3605.38			
27	3718.10	3718.66	3719.22	$M_{3,19}$	3700±200 Hendry	78
28	3832.50	3833.04	3833.58			
29	3947.40	3947.93	3948.45			
30	4062.77	4063.27	4063.78	$N_{3,21}$	4100±300 Hendry	78

Table 10. Kaluza-Klein tower of KK excitations for the $N\rho$ system and N, Δ baryons.

n	$M_n^{n\rho}$ MeV	$M_n^{p\,\rho}$ MeV	M_{exp}^{N} MeV	M_{exp}^{Δ} MeV
1	1710.90	1709.61	N[1710]P_{11}	$D_{33}(1710\pm30)$
2	1716.98	1715.69	$P_{11}(1717\pm28)$	
3	1727.07	1725.79	N[1720]P_{13}	
4	1741.09	1739.82		$P_{31}(1744\pm36)$
5	1758.95	1757.69	$P_{11}(1766\pm34)$	Δ[1750]P_{31}
				$F_{35}(1752\pm32)$
6	1780.53	1779.28		
7	1805.69	1804.45	$D_{13}(1804\pm55)$	$S_{31}(1802\pm87)$
8	1834.27	1833.05		
9	1866.12	1864.92	$P_{13}(1879\pm17)$	$S_{31}(1890\pm50)$
10	1901.07	1899.89	N[1900]P_{13}	Δ[1900]S_{31}
				Δ[1905]F_{35}
				Δ[1910]P_{31}
				Δ[1920]P_{33}
				Δ[1930]D_{35}
11	1938.94	1937.78		Δ[1940]D_{33}
				Δ[1950]F_{37}
			N[1990]F_{17}	
12	1979.58	1978.43	$F_{17}(1970\pm50)$	$F_{37}(1950\pm15)$
13	2022.80	2021.68	N[2000]F_{15}	Δ[2000]F_{35}
			$F_{15}(\approx2025)$	$P_{33}(2014\pm16)$
14	2068.44	2067.34	N[2080]D_{13}	$D_{33}(2057\pm110)$
			N[2090]S_{11}	
15	2116.35	2115.27	N[2100]P_{11}	
			N[2190]G_{17}	Δ[2150]S_{31}
16	2166.37	2165.31	$G_{17}(2168\pm18)$	$D_{35}(2171\pm18)$
			$S_{11}(2180\pm80)$	$S_{31}(2150\pm100)$
			N[2200]D_{15}	Δ[2200]G_{37}
17	2218.36	2217.33	$D_{15}(2228\pm30)$	$G_{37}(2215\pm60)$
			N[2220]H_{19}	$H_{39}(2217\pm80)$
18	2272.19	2271.17	N[2250]G_{19}	
19	2327.72	2326.73	$H_{19}(2300\pm100)$	Δ[2300]H_{39}
				Δ[2350]D_{35}
20	2384.83	2383.86		Δ[2390]F_{37}
				Δ[2400]G_{39}
21	2443.43	2442.48		Δ[2420]$H_{3,11}$
				$G_{39}(2468\pm50)$
22	2503.39	2502.46		$H_{39}(2450\pm100)$
23	2564.63	2563.72	$I_{1,11}(2577\pm50)$	
24	2627.06	2626.17	$K_{1,13}(2612\pm45)$	$I_{3,13}(2650\pm100)$
25	2690.58	2689.72	$I_{1,11}(2700\pm100)$	
26	2755.14	2754.29		Δ[2750]$I_{3,13}$
27	2820.66	2819.83		$I_{3,13}(2794\pm80)$
28	2887.06	2886.25		$K_{3,15}(2850\pm100)$
29	2954.31	2953.51		Δ[2950]$K_{3,15}$
30	3022.32	3021.54	$K_{1,13}(3000\pm100)$	

Table 11. Kaluza-Klein tower of KK excitations for the $N\eta$ system and N, Δ baryons.

n	$M_n^{n\eta}$ MeV	$M_n^{p\eta}$ MeV	M_{exp}^{N} MeV	M_{exp}^{Δ} MeV
1	1489.35	1488.06		
2	1496.77	1495.48		
3	1509.04	1507.76		
4	1526.00	1524.73	N[1520]D_{13}	
5	1547.47	1546.20	N[1535]S_{11}	
6	1573.20	1571.95		
7	1602.97	1601.73		Δ[1600]P_{33}
8	1636.50	1635.28	N[1650]S_{11}	Δ[1620]S_{31}
9	1673.52	1672.32	N[1675]D_{15}	
			N[1680]F_{15}	
			N[1700]D_{13}	Δ[1700]D_{33}
10	1713.80	1712.61	N[1710]P_{11}	
			N[1720]P_{13}	
11	1757.06	1755.90	P_{11}(1766±34)	Δ[1750]P_{31}
12	1803.09	1801.94	D_{13}(1804±55)	S_{31}(1802±87)
13	1851.65	1850.53		
14	1902.55	1901.45	N[1900]P_{13}	Δ[1900]S_{31}
				Δ[1905]F_{35}
				Δ[1910]P_{31}
				Δ[1920]P_{33}
				Δ[1930]D_{35}
				Δ[1940]D_{33}
15	1955.58	1954.51	N[1990]F_{17}	Δ[1950]F_{37}
16	2010.59	2009.54	N[2000]F_{15}	Δ[2000]F_{35}
17	2067.40	2066.37	N[2080]D_{13}	D_{33}(2057±110)
			N[2090]S_{11}	
18	2125.88	2124.87	N[2100]P_{11}	Δ[2150]S_{31}
19	2185.89	2184.90	N[2190]G_{17}	D_{35}(2171±18)
			N[2200]D_{15}	Δ[2200]G_{37}
20	2247.31	2246.34	N[2220]H_{19}	H_{39}(2217±80)
			N[2250]G_{19}	
21	2310.02	2309.07	H_{19}(2300±100)	Δ[2300]H_{39}
				D_{35}(2305±26)
22	2373.93	2373.00		Δ[2390]F_{37}
23	2438.94	2438.03		Δ[2400]G_{39}
				Δ[2420]$H_{3,11}$
24	2504.98	2504.09		H_{39}(2450±100)
25	2571.95	2571.09	$I_{1,11}$(2577±50)	
26	2639.81	2638.96	$K_{1,13}$(2612±45)	$I_{3,13}$(2650±100)
27	2708.47	2707.64	$I_{1,11}$(2700±100)	
28	2777.89	2777.07		Δ[2750]$I_{3,13}$
29	2848.00	2847.20		$K_{3,15}$(2850±100)
30	2918.77	2917.99		Δ[2950]$K_{3,15}$

Table 12. Kaluza-Klein tower of KK excitations for the $N\omega$ system and N, Δ baryons.

n	$M_n^{n\omega}$ MeV	$M_n^{p\omega}$ MeV	M_{exp}^{N} MeV	M_{exp}^{Δ} MeV
1 2 3	1724.15 1730.18 1740.17	1722.86 1728.89 1738.89	N[1720]P_{13}	D_{33}(1710±30) P_{31}(1744±36)
4	1754.07	1752.80		Δ[1750]P_{31} F_{35}(1752±32)
5 6 7 8 9	1771.77 1793.17 1818.11 1846.47 1878.08	1770.51 1791.92 1816.88 1845.25 1876.88	P_{11}(1766±34) D_{13}(1804±55) P_{13}(1879±17)	 S_{31}(1802±87) S_{31}(1890±50)
10	1912.77	1911.59	N[1900]P_{13}	Δ[1900]S_{31} Δ[1905]F_{35} Δ[1910]P_{31} Δ[1920]P_{33}
11	1950.38	1949.22		Δ[1930]D_{35} Δ[1940]D_{33} Δ[1950]F_{37}
12	1990.75	1989.60	N[1990]F_{17} F_{17}(1970±50)	 F_{37}(1950±15)
13	2033.69	2032.57	N[2000]F_{15} F_{15}(≈2025)	Δ[2000]F_{35} P_{33}(2014±16)
14	2079.06	2077.96	N[2080]D_{13} N[2090]S_{11}	D_{33}(2057±110)
15	2126.70	2125.62	N[2100]P_{11}	
16	2176.45	2175.40	N[2190]G_{17} G_{17}(2168±18) S_{11}(2180±80)	Δ[2150]S_{31} D_{35}(2171±18) S_{31}(2150±100)
17	2228.18	2227.15	N[2200]D_{15} D_{15}(2228±30) N[2220]H_{19}	Δ[2200]G_{37} G_{37}(2215±60) H_{39}(2217±80)
18	2281.75	2280.74	N[2250]G_{19}	
19	2337.03	2336.04	H_{19}(2300±100)	Δ[2300]H_{39} Δ[2350]D_{35}
20	2393.90	2392.93		Δ[2390]F_{37} Δ[2400]G_{39}
21	2452.25	2451.30		Δ[2420]$H_{3,11}$ G_{39}(2468±50)
22 23 24 25 26 27 28 29 30	2511.99 2573.00 2635.21 2698.53 2762.89 2828.21 2894.44 2961.50 3029.34	2511.06 2572.10 2634.32 2697.67 2762.04 2827.38 2893.62 2960.70 3028.56	 $I_{1,11}$(2577±50) $K_{1,13}$(2612±45) $I_{1,11}$(2700±100) $K_{1,13}$(3000±100)	H_{39}(2450±100) $I_{3,13}$(2650±100) Δ[2750]$I_{3,13}$ $I_{3,13}$(2794±80) $K_{3,15}$(2850±100) Δ[2950]$K_{3,15}$

Table 13. Kaluza-Klein tower of KK excitations for the ΛK system and N, Δ baryons.

n	$M_n^{\Lambda K^\pm}$ MeV	$M_n^{\Lambda K^0}$ MeV	M_{exp}^{N} MeV	M_{exp}^{Δ} MeV
1	1611.87	1615.85		$\Delta[1600]P_{33}$
2	1619.36	1623.30		$\Delta[1620]S_{31}$
3	1631.72	1635.60		
4	1648.77	1652.56	N$[1650]S_{11}$	
5	1670.27	1673.96	N$[1675]D_{15}$	
			N$[1680]F_{15}$	
6	1695.97	1699.54	N$[1700]D_{13}$	$\Delta[1700]D_{33}$
7	1725.59	1729.04	N$[1710]P_{11}$	
			N$[1720]P_{13}$	
8	1758.84	1762.16	$P_{11}(1766\pm34)$	$\Delta[1750]P_{31}$
9	1795.43	1798.62	$D_{13}(1804\pm55)$	$S_{31}(1802\pm87)$
10	1835.12	1838.18		
11	1877.64	1880.57	$P_{13}(1879\pm17)$	
12	1922.76	1925.58	N$[1900]P_{13}$	$\Delta[1900]S_{31}$
				$\Delta[1905]F_{35}$
				$\Delta[1910]P_{31}$
				$\Delta[1920]P_{33}$
				$\Delta[1930]D_{35}$
				$\Delta[1940]D_{33}$
13	1970.28	1972.98	N$[1990]F_{17}$	$\Delta[1950]F_{37}$
14	2019.99	2022.59	N$[2000]F_{15}$	$\Delta[2000]F_{35}$
15	2071.74	2074.22	N$[2080]D_{13}$	$D_{33}(2057\pm110)$
			N$[2090]S_{11}$	
16	2125.34	2127.73	N$[2100]P_{11}$	$\Delta[2150]S_{31}$
17	2180.67	2182.97	N$[2190]G_{17}$	$D_{35}(2171\pm18)$
			N$[2200]D_{15}$	$\Delta[2200]G_{37}$
18	2237.59	2239.80	N$[2220]H_{19}$	$H_{39}(2217\pm80)$
			N$[2250]G_{19}$	
19	2295.98	2298.11	$H_{19}(2300\pm100)$	$\Delta[2300]H_{39}$
20	2355.73	2357.78		$\Delta[2350]D_{35}$
				$F_{37}(2350\pm100)$
21	2416.75	2418.72		$\Delta[2400]G_{39}$
				$\Delta[2420]H_{3,11}$
22	2478.94	2480.84		$H_{39}(2450\pm100)$
23	2542.22	2544.06	$I_{1,11}(2577\pm50)$	
24	2606.51	2608.29	$K_{1,13}(2612\pm45)$	
25	2671.76	2673.48	$I_{1,11}(2700\pm100$	$I_{3,13}(2650\pm100)$
26	2737.88	2739.55		$\Delta[2750]I_{3,13}$
27	2804.83	2806.45		$I_{3,13}(2794\pm80)$
28	2872.56	2874.13		$K_{3,15}(2850\pm100)$
29	2941.00	2942.52		$\Delta[2950]K_{3,15}$
30	3010.12	3011.60	$K_{1,13}(3000\pm100)$	

Table 14. KK excitations for the $(\Sigma K)^{\pm}$ system and N, Δ baryons.

n	$M_n^{\Sigma^+ K^0}$MeV	$M_n^{\Sigma^- K^0}$MeV	$M_n^{\Sigma^0 K^{\pm}}$MeV	M_{exp}^{N} MeV	M_{exp}^{Δ} MeV
1	1689.49	1697.59	1688.78	N[1680]F_{15}	
2	1696.80	1704.88	1696.12	N[1700]D_{13}	Δ[1700]D_{33}
3	1708.86	1716.91	1708.24	N[1710]P_{11}	
4	1725.49	1733.51	1724.94	N[1720]P_{13}	
5	1746.48	1754.46	1746.02		Δ[1750]P_{31} $F_{35}(1752\pm32)$
6	1771.57	1779.50	1771.20	$P_{11}(1766\pm34)$	
7	1800.49	1808.36	1800.22	$D_{13}(1804\pm55)$	$S_{31}(1802\pm87)$
8	1832.96	1840.77	1832.80		
9	1868.72	1876.45	1868.65	$P_{13}(1879\pm17)$	
10	1907.51	1915.16	1907.54	N[1900]P_{13}	Δ[1905]F_{35} Δ[1900]S_{31} Δ[1910]P_{31} Δ[1920]P_{33}
11	1949.08	1956.65	1949.20		Δ[1930]D_{35} Δ[1940]D_{33} Δ[1950]F_{37}
12	1993.22	2000.70	1993.42	N[1990]F_{17}	$P_{33}(2014\pm16)$
13	2039.72	2047.10	2039.99	N[2000]F_{15}	Δ[2000]F_{35}
14	2088.39	2095.67	2088.73	N[2080]D_{13} N[2090]S_{11} N[2100]P_{11}	$D_{33}(2057\pm110)$
15	2139.06	2146.24	2139.47		Δ[2150]S_{31}
16	2191.58	2198.66	2192.05	N[2190]G_{17} N[2200]D_{15}	Δ[2200]G_{37} $D_{35}(2171\pm18)$
17	2245.82	2252.79	2246.34	N[2220]H_{19} N[2250]G_{19}	
18	2301.64	2308.50	2302.20	$H_{19}(2300)$	Δ[2300]H_{39}
19	2358.92	2365.68	2359.53		Δ[2350]D_{35}
20	2417.58	2424.23	2418.21		Δ[2390]F_{37} Δ[2400]G_{39} Δ[2420]$H_{3,11}$
21	2477.50	2484.04	2478.17		$H_{39}(2450\pm100)$
22	2538.61	2545.04	2539.30		
23	2600.81	2607.14	2601.53	N[2600]$I_{1,11}$	
24	2664.05	2670.27	2664.78		$I_{3,13}(2650\pm100)$
25	2728.25	2734.36	2728.99	N[2700]$K_{1,13}$	
26	2793.35	2799.36	2794.10		Δ[2750]$I_{3,13}$
27	2859.29	2865.20	2860.06		$K_{3,15}(2850\pm100)$
28	2926.02	2931.83	2926.80		Δ[2950]$K_{3,15}$
29	2993.49	2999.20	2994.27	$K_{1,13}(3000)$	$K_{3,15}(2990\pm100)$
30	3061.66	3067.27	3062.45		

Table 15. KK excitations for the $(\Sigma K)^{0,++,--}$ system and N, Δ baryons.

n	$M_n^{\Sigma^+ K^\pm}$MeV	$M_n^{\Sigma^- K^\pm}$MeV	$M_n^{\Sigma^0 K^0}$MeV	M_{exp}^NMeV	M_{exp}^ΔMeV
1 2 3 4	1685.51 1692.86 1704.98 1721.70	1693.60 1700.94 1713.04 1729.73	1692.76 1700.06 1712.11 1728.73	N[1680]F_{15} N[1700]D_{13} N[1710]P_{11} N[1720]P_{13}	Δ[1700]D_{33}
5	1742.79	1750.78	1749.70		Δ[1750]P_{31} $F_{35}(1752\pm32)$
6 7 8 9	1768.00 1797.04 1829.64 1865.53	1775.93 1804.91 1837.45 1873.26	1774.77 1803.67 1836.12 1871.84	$P_{11}(1766\pm34)$ $D_{13}(1804\pm55)$ $P_{13}(1879\pm17)$	$S_{31}(1802\pm87)$
10	1904.45	1912.10	1910.60	N[1900]P_{13}	Δ[1905]F_{35} Δ[1900]S_{31} Δ[1910]P_{31} Δ[1920]P_{33}
11	1946.14	1953.71	1952.14		Δ[1930]D_{35} Δ[1940]D_{33} Δ[1950]F_{37}
12 13	1990.40 2037.01	1997.88 2044.40	1996.24 2042.70	N[1990]F_{17} N[2000]F_{15}	$P_{33}(2014\pm16)$ Δ[2000]F_{35}
14	2085.79	2093.08	2091.33	N[2080]D_{13} N[2090]S_{11} N[2100]P_{11}	$D_{33}(2057\pm110)$
15	2136.57	2143.75	2141.96		Δ[2150]S_{31}
16	2189.19	2196.27	2194.44	N[2190]G_{17} N[2200]D_{15}	Δ[2200]G_{37} $D_{35}(2171\pm18)$
17	2243.52	2250.49	2248.63	N[2220]H_{19} N[2250]G_{19}	
18 19	2299.43 2356.80	2306.29 2363.56	2304.41 2361.65	$H_{19}(2300)$	Δ[2300]H_{39} Δ[2350]D_{35}
20	2415.53	2422.18	2420.26		Δ[2390]F_{37} Δ[2400]G_{39} Δ[2420]$H_{3,11}$
21 22 23 24 25 26 27 28 29 30	2475.52 2536.70 2598.97 2662.27 2726.53 2791.68 2857.67 2924.45 2991.97 3060.18	2482.07 2543.13 2605.30 2668.49 2732.64 2797.69 2863.58 2930.26 2997.68 3065.79	2480.14 2541.20 2603.37 2666.56 2730.72 2795.77 2861.67 2928.36 2995.80 3063.93	 N[2600]$I_{1,11}$ N[2700]$K_{1,13}$ $K_{1,13}(3000)$ 	$H_{39}(2450\pm100)$ $I_{3,13}(2650\pm100)$ Δ[2750]$I_{3,13}$ $K_{3,15}(2850\pm100)$ Δ[2950]$K_{3,15}$ $K_{3,15}(2990\pm100)$